# Simulation of the acoustic field produced by cavities using the Boundary Element – Rayleigh Integral Method (BERIM) and its application to a horn loudspeaker.

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## Introduction

In this paper a method based on coupling the interior boundary element method (BEM) and the Rayleigh Integral Method (RIM) for simulating the acoustic field of a cavity with one opening is proposed. Such a method has a number of applications in acoustics. In this paper we will be applying the method to the problem of determining the acoustic response of a typical horn loudspeaker.

Figure 1 W8LC Line Array, box and horn element. Highlighted section modelled.

In order to couple the BEM and RIM methods, a fictitious boundary is placed across the opening of the cavity. The interior boundary integral equation formulation of the cavity is coupled to the Rayleigh integral which governs the exterior acoustic field, by enforcing continuity in the velocity potential (sound pressure) and its derivative on the opening. The method is termed the Boundary Element –Rayleigh Integral Method (BERIM) and it only requires a mesh of the interior cavity and the opening, and hence it is generally much more efficient than the straightforward application of the exterior BEM to this kind of problem. The coupling of the integral equations gives a linear system of equations, the solution of which returns the velocity potential (sound pressure) and its normal derivative (velocity) on the cavity surface and on the opening. The acoustic properties can then be found either within the cavity using the interior boundary integral equation formulation or in the exterior region by using the Rayleigh integral. The BERIM method is developed in 3D through a simple triangulation of the cavity and the opening and the application of collocation to the integral equations to give the Fortran subroutine BERIM3. In this paper the BERIM3 method is verified by applying it to the horn loudspeaker illustrated in Figure 1 and comparing the results with results from the exterior BEM.

A horn loudspeaker is a type of acoustic transducer which presents to the vibrating piston a higher acoustic resistance than experienced by a piston in free air. The shape of the horn controls the degree of loading and directional characteristics. Practical horns do not generally conform to the classical flares – for example exponential or hyperbolic – but are formed from geometry which prevents simple analysis. In professional sound reinforcement horns have been an essential feature for many years, Martin Audio (2004). Amongst its virtues are higher efficiency and a control over directional characteristics. The latter has become very important in recent years, due to the advent of high power amplifiers and compression drivers built to withstand them. It is for this reason that we concentrate on the SPL and polar response in this paper; results from BERIM3 applied to the horn loudspeaker of Figure 1 are presented for a wide range of sample frequencies.

## Boundary Element- Rayleigh Integral Method (BERIM) Model

There are at least three approaches to solving the open cavity problem using integral equation techniques. One method is to treat it as an exterior problem and apply the BEM by wrapping elements both around the exterior and the interior cavity walls, for example by using the AEBEM* methods Kirkup (2004a). A second method is to close the cavity and couple boundary integral equation reformulations of the interior and exterior regions across the openings (eg coupling the AIBEM* and AEBEM* programs of Kirkup (2004a)). An alternative method is to close the (one) opening of the cavity and couple the interior boundary integral equation with the Rayleigh integral (ie coupling the AIBEM* and ARIM* methods of Kirkup (2004a,b)]. It is this the third idea, boundary element- Rayleigh integral method (BERIM) that we will be developing in this paper.

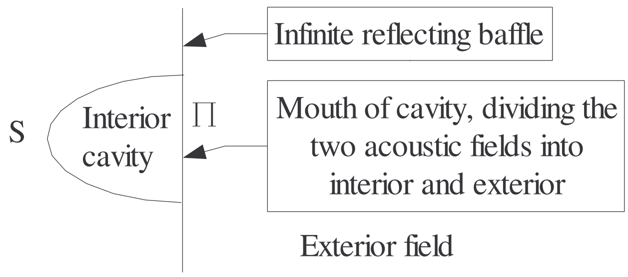



Figure 2. Preparation of model for application of BERIM.

The physical problem is now illustrated by Figure 2. The acoustic domain is the cavity and the half-space beyond the mouth. The baffle is rigid and perfectly reflecting. This model can be applied to a range of acoustic cavity problems. In any practical problem the baffle must be finite but, even if there is no baffle, at least the continuity in the acoustic field is maintained across the mouth and the model can still be applied with due care. In the case of a horn loudspeaker, such as the one illustrated in Figure 1, there is a substantial baffle and the model is considered very appropriate.

Let S be the surface of the cavity and Π be the opening. The boundary condition is applied on the surface of the cavity and the condition is presumed to be in the following form:

$$a(\mathbf{p})\varphi(\mathbf{p})+b(\mathbf{p})v(\mathbf{p})=f(\mathbf{p}) \qquad (\mathbf{p}\in \mathrm{S}), \tag{1}$$

though for the horn loudspeaker application in this work only the Neumann condition is considered: $a(\boldsymbol{p})=0$, $b(\boldsymbol{p})=1$ ($\mathbf{p}\in$ S).

## BERIM Method

The BERIM method is derived through coupling the interior boundary integral equation formulation for points on interior cavity surface:

$$\{M_k+\frac{1}{2}I\}_{S+\Pi}\,\varphi(\mathbf{p})=\{L_k\}_{S+\Pi}\,v(\mathbf{p}) \quad (\mathbf{p}\in \mathrm{S+\Pi}) \tag{2}$$

and the Rayleigh integral for points on Π,

$$v(\mathbf{p})=-2\{L_k\}_{\Pi}\,\varphi(\mathbf{p}) \quad (\mathbf{p}\in \Pi). \tag{3}$$

In equations (2) and (3),φ represents the velocity potential and $v$ its derivative with respect to the normal that is outward to the cavity. The operators are defined as follows:

$$\{L_k\mu\}_{\Gamma}(\mathbf{p})=\int_{\Gamma}G(\mathbf{p},\mathbf{q})\mu(\mathbf{q})dS_q \text{ and } \{M_k\mu\}_{\Gamma}(\mathbf{p})=\int_{\Gamma}\frac{\partial G(\mathbf{p},\mathbf{q})}{\partial n_q}\mu(\mathbf{q})dS_q$$

where G is the free-space Green's function for the Helmholtz equation and Γ is used here to represent any surface or part of the surface (including Π), $I$ is the identity operator.

If we consider equation (3) for points on S and Π separately then we obtain the following equations:

$$\{M_k + \frac{1}{2}I\}_S\, \varphi(\mathbf{p}) + \{M_k\}_\Pi\, \varphi\ (\mathbf{p}) = \{L_k\}_S\, v(\mathbf{p}) + \{L_k\}_\Pi\, v\ (\mathbf{p}) \quad (\mathbf{p} \in \mathrm{S}) \tag{4}$$

$$\{M_k\}_{S+\Pi}\, \varphi(\mathbf{p}) + \{M_k + \frac{1}{2}I\}_\Pi\, \varphi(\mathbf{p}) = \{L_k\}_S\, v(\mathbf{p}) + \{L_k\}_\Pi\, v(\mathbf{p}) \quad (\mathbf{p} \in \Pi) \tag{5}$$

The computational method is applied by a triangulation of the interior surface of the cavity and the opening alone. By approximating $\varphi$ and $v$ by constants on each triangle and through collocation the integral equations (3),(4), and (5) can be written as linear systems of equations:

$$\underline{v}_\Pi = -2[L_k]_{\Pi\Pi}\, \underline{\varphi}_\Pi\,, \tag{6}$$

$$[M_k + \frac{1}{2}I]_{SS}\, \underline{\varphi}_S + [M_k]_{S\Pi}\, \underline{\varphi}_\Pi = [L_k]_{SS}\, \underline{v}_S + [L_k]_{S\Pi}\, \underline{v}_\Pi\,, \tag{7}$$

$$[M_k]_{\Pi S}\, \underline{\varphi}_S + [M_k + \frac{1}{2}I]_{\Pi\Pi}\, \underline{\varphi}_\Pi = [L_k]_{\Pi S}\, \underline{v}_S + [L_k]_{\Pi\Pi}\, \underline{v}_\Pi\,, \tag{8}$$

respectively. The correspondence between (3-5) and (6-8) should be clear. The operators $L_k$, $M_k$ and $I$ are replaced by the matrices $[L_k]$,$[M_k]$ and $[I]$ and the boundary functions $\varphi$ and $v$ are replaced by vectors $\underline{\varphi}$ and $\underline{v}$. For more details on this see Kirkup (2004a).

In the collocation method the centres of the triangles, the collocation points, are the representative points on the cavity surface and opening at which the surface functions $\varphi$ and $v$ are observed. If the cavity surface S is divided into n elements and opening Π is divided into m elements then $\underline{\varphi}_S$ is an n-vector, $\underline{\varphi}_\Pi$ is an m-vector, $[L_k]_{S\Pi}$ is an nxm matrix etc. With equations (6-8) we then have n+2m equations with potentially 2n+2m unknowns. The system is completed with the n equations that are provided by the discrete form of the boundary condition (1):

$$[D_a]_{SS}\, \underline{\varphi}_S + [D_b]_{SS}\, \underline{v}_S = \underline{f}_S \tag{9}$$

where $[D_a]_{SS}$ and $[D_b]_{SS}$ are diagonal nxn matrices with the diagonal made up of the values of $a(\mathbf{p})$ and b($\mathbf{p}$) at the collocation points on S.

Using equations (6-9) we can form a (2n+2m)x(2n+2m) system of equations that returns approximations to the values of $\varphi$ and $v$ at the collocation points. For purely Neumann or Dirichlet boundary conditions we can simplify (9) and in these cases we can write the coupled system as an (n+2m)x(n+2m) system, this simplification is made in BERIM3. Once the surface functions are determined, results on the cavity D can be found using the integral

$$\varphi(\mathbf{p}) = \{L_k\}_{S+\Pi} v(\mathbf{p}) - \{M_k\}_{s+\Pi} \varphi(\mathbf{p}) \quad (\mathbf{p} \in \mathrm{D}), \tag{10}$$

and the Rayleigh integral can be used again for points in the exterior E

$$v(\mathbf{p}) = -2\{L_k\}_{\Pi} \varphi(\mathbf{p}) \quad (\mathbf{p} \in \mathrm{E}). \tag{11}$$

## Application of BERIM3 to the Horn Loudspeaker

In order to apply BERIM3 to the horn loudspeaker shown in figure 1, first the 3D solid model is generated automatically from a set of around 10 parameters. This is then introduced into the popular GID pre/post processor where a triangulation of the interior surface and mouth is made and subsequently solved. A typical GID post process mesh is shown in figure 3. A velocity of 1m/s was set at the "throat" (assumed to be flat) and zero everywhere else. In order to mitigate the numerical effects of the sudden change in boundary conditions where the cavity surface meets the mouth, a small flange was added. A description of each calculation can be found in Table 1, where number of elements and approximate running time on a AMD2200 PC platform are given.

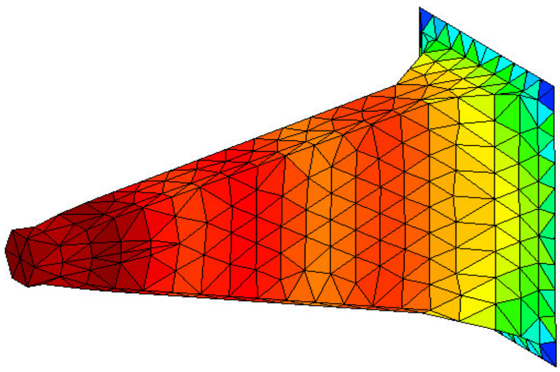
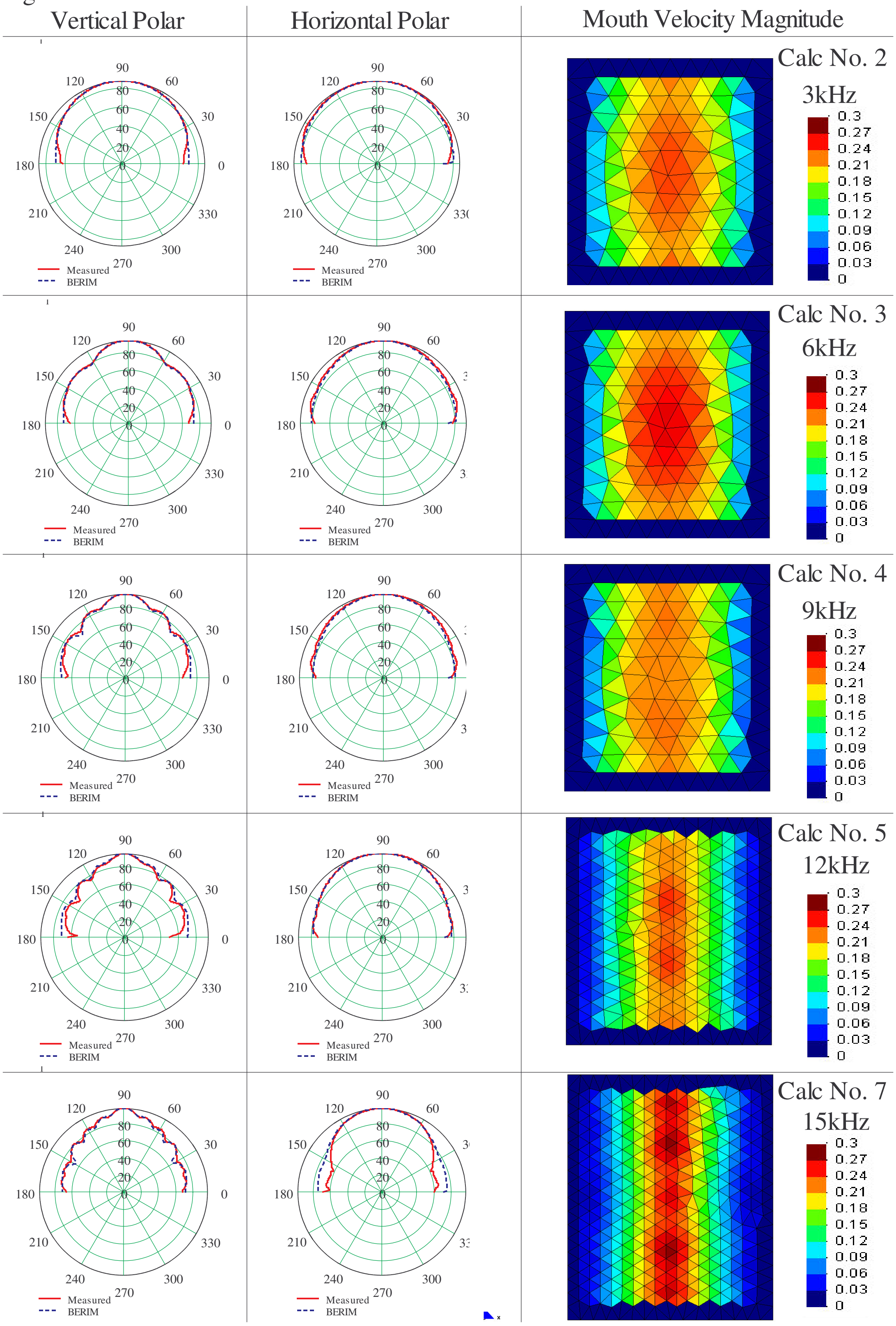


Figure 3 Typical BERIM3 mesh showing surface SPL at 3kHz

The sound pressure is observed on polar paths of 1m radius. The results from BERIM3 are compared with measured results in Figure 4, showing polar plots of the sound pressure level (spl) in the vertical and horizontal polar plane and an illustration of the mouth velocity amplitude for 3,6,9,12,and 15kHz. The popular GID pre/post processor was used to mesh and display the results.

Fig 4 Results



## Comparison of BERIM3 with BEM

By way of comparison and further validation, the application of BERIM3 is compared with the application of the boundary element method (AEBEM3) to the same problem, but at 3kHz only. In order to apply the BEM, the mesh in Figure 5 is used. The horizontal and vertical polar plots of the SPL at 1m is shown in figure 6.

Fig 5 AEBEM Mesh
SPL

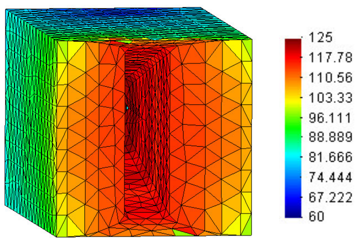
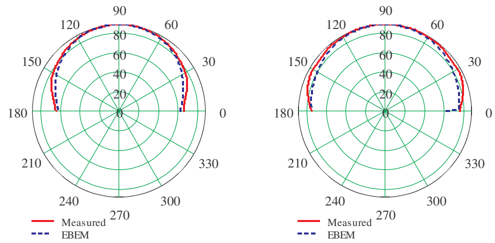



Fig 6 AEBEM Results

Vertical Polar    Horizontal Polar




| Calculation | Solver | Freq | Element Size | Num Elements | Time |
|---|---|---|---|---|---|
| 1 | AEBEM | 3kHz | 12mm | 2189 | 23min |
| 2 | BERIM | 3kHz | 12mm | 994 | 2min |
| 3 | BERIM | 6kHz | 12mm | 994 | 2min |
| 4 | BERIM | 9kHz | 12mm | 994 | 2min |
| 5 | BERIM | 12kHz | 8mm | 2035 | 25min |
| 6 | BERIM | 12kHz | 7mm | 2600 | 56min |
| 7 | BERIM | 15kHz | 7mm | 2600 | 56min |

Table1 Computation Timings

## Conclusion

For a structure such as a horn loudspeaker, which consists of a cavity (the horn) opening out on to a plane, the Boundary Element – Rayleigh Integral Method (BERIM) seems most applicable. In Figure 3 it is shown that BERIM requires a mesh of the interior surface and opening plane alone whereas the application of the boundary element method (BEM) to the same problem requires considerably more elements. Hence when it can be applied BERIM3 reduces the meshing required and typically uses an order of magnitude less computer time than the straightforward BEM.

The results in Figure 6, compared with Figure 4 at 3kHz show good agreement between computed and measured results, there are a number of other points. BERIM3 seems to give better agreement with measured than the BEM in the forward field, however, near the baffle the BEM has more agreement. The proposed reason for this is that the BEM accurately meshes the baffle whereas BERIM assumes and infinite baffle; BERIM3 gives more support to the wider field than the true finite baffle.

Taking into account the comment in the previous paragraph on the modelling of the wider field, the results generally show good agreement between measured and computed in Figure 4. In generally the lobes in the sound field are captured. There is only significant drift in the horizontal polar at 15kHz: this would probably benefit from a further refinement in the mesh. The present method represents a significant improvement over our initial acoustic models Webb, Baird (2003). In general BERIM3 is a powerful tool for the simulation of the sound field of a horn loudspeaker; returning results for a given problem and given frequency within a few minutes at low and medium frequencies on a typical modern PC.

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